Incorporating uncertainties into economic forecasts: an application to forecasting economic activity in Croatia

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Abstract

In this paper we present a framework for incorporating uncertainties into economic activity forecasts for Croatia. Using the vector error correction model (VECM) proposed by Rukelj (2010) as the benchmark model, we forecast densities of the variable of interest using stochastic simulations for incorporating future and parameter uncertainty. We exploit the use of parametric and non-parametric approaches in generating random shocks as in Garrat et al. (2003). Finally we evaluate the results by the Kolmogorov-Smirnov and Anderson-Darling test of probability integral transforms. The main findings are: (1) the parametric and the non-parametric approach yield similar results; (2) the incorporation of parameter uncertainty results in much wider probability forecast; and (3) evaluation of density forecasts indicates better performance when only future uncertainties are considered and parameter uncertainties are excluded.

Keywords: economic forecasting, density forecasting, fan chart, stochastic simulations, uncertainty, Croatia

1 INTRODUCTION

Economic forecasting has been one of the most recurrent topics for policy makers. Whilst its relevance for policy decisions is unquestioned, the failures and inaccuracies in forecasting witnessed across time have led to a long debate. This debate includes examining predictive power, the evaluation of forecasts and the use of econometric tools when analyzing macroeconomic behavior and its projections. In the literature it is possible to identify two main strands concerning economic forecasting. Firstly, that which investigates whether to use point predictions or probabilities[1]. Probabilities lead to the second strand, which rationalizes the use of probability measures as a way of incorporating uncertainties[2]. Particularly, Garrat et al. (2003) notice that in general macroeconomic forecasts are presented as point forecasts, and when uncertainty is considered, uncertainty being characterised by confidence intervals. As recently pointed out by Engelberg et al. (2009), the incorporation of probabilities in forecasting can improve the interpretation of point predictions[3].

[1] Juster (1966) for example notices that a probability variable in consumer's surveys could more accurately predict purchase rates if compared to the predictors rising from a point projection of buying intentions. Although quite specific, the problem of accuracy in predicting the demand for durable goods by using such surveys is addressed in a way that concludes by fitting in a more general issue in statistics: whether to incorporate probabilities in the forecasts or remain with point predictions. In fact, Zarnowitz and Lambros (1987) compare consensus (point prediction) and uncertainty (diffuseness of the corresponding probability distributions), aiming to answer whether the dispersion of the point forecasts reflects lack of confidence of the correspondent predictor.

[2] A comprehensive survey of applications of density forecasting until late the 1990s is done by Tay and Wallis (2000).

[3] For example, probability forecasts using macroeconomic models are proposed by Fair (1980) for the US. This study accounts for uncertainty about the error terms, exogenous variable forecasts, the parameters and the possible misspecification of the model. It also considers the fact that the variances of forecast errors are not constant across time. Doan et al. (1984) choose the Bayesian approach in estimating an unrestricted and time-varying vector autoregressive processes for its forecasting exercise, showing that it improves the out-of-sample forecasts relative to univariate equations (by basically reporting a weighted likelihood by priors,

If literature has reached a consensus, it is that uncertainties have to be incorporated into the forecasting framework. However this is not trivial, as, depending on the source of uncertainty – about the future, the parameters of the considered model, or the model itself – the forecaster has to evaluate different ways of taking each type of uncertainty into account. Clements and Hendry (1995) examine the frameworks for economic forecasting. Basically, the authors notice that in the forecasting context, the methods for forecasting models and procedures might not be the only source of failures, which might also involve the states of nature related to the properties of the variables to be forecasted. They argued that the assumptions of constant, time-invariant and stationary data generating processes, also thought to be coincident with a unique model of the economy, were a wrong way of representing the world.

Practically, it has been observed that in the inflation targeting policies of many countries the incorporation of uncertainty in economic variable predictions has served to show that there is uncertainty about shocks affecting the economy and about the nature of the transmission mechanism; it has also helped to communicate with as minimum ambiguity as possible the views of the economic policy authority; and to provide a better understanding of the sources of uncertainty (Blix and Sellin, 1998).

Methods for the incorporation of uncertainty are not uniform. As shown by Britton et al. (1998), the proposed model of which is followed by the Bank of England for inflation and output growth forecasts, the early ranges of uncertainty in projections based on historical forecast errors were not as satisfactory as had been expected, because, by their construction, they did not allow for asymmetries, thus not considering alternative scenarios and not enabling any conclusions about the risk views. If centered, the forecast would represent the upper and lower boundaries, rather than probabilities. Given this drawback, the Bank of England's forecasting method started considering probability distributions by assuming it had a known functional form and evaluating a limiting number of alternative assumptions. Another example of a practical use of density forecasting is the Sveriges Riskbank's forecasting method, which is based on Blix and Sellin (1998), who emphasize the important implications of slightly changing the model proposed by Britton et al. (1998). The change consists of determining the balance of risks by subjective assessments of the macroeconomic variables of which uncertainty is considered.

In this paper, we study whether interesting and useful results would arise from applying such methods in forecasting economic activity in Croatia. Such an approach would be more appropriate than point forecasts, when we are considering the impact of large shocks in the economy, such as the last financial crisis was. We

unlike the non weighted likelihood coming from other alternatives), and explaining how the model could be used to make conditional projections and to analyze policy alternatives.

propose density forecasts which can cover the whole set of possible outcomes with probabilities of their realization and in this way account for uncertainty in forecasting. The importance of the paper inheres in its presentation of density forecasts of economic activity for Croatia for the first time. For the incorporation of uncertainties, we consider the approach proposed by Garrat et al. (2003), in which they evaluate probability forecasts in the context of a small long-run structural vector error correction autoregressive model (VECM) of the UK economy. In our case, the analysis is done using a slight modification of the model proposed by Rukelj (2010) for the Croatian economy.

The remainder of this paper is as follows: in the next section, we present the data and its features, as well as the benchmark model considered and an analysis of its forecasting performance.

In Section 3 the methodology is explained and there is a detailed account of incorporation of uncertainty about both the future and about the future and the parameters, where parametric and non-parametric approaches for stochastic simulation of shocks are presented together with risk asymmetries. Within the same section we provide a description and elaboration of the fan charts and forecast evaluation. We present the results in Section 4 and a further discussion in Section 5, to conclude in section 6.

2 DATA AND THE BENCHMARK MODEL

Forecasting uncertainty in this paper is examined with the use of an estimated econometric model, which we refer to as the benchmark model in the rest of the paper. Three dimensional VECM was chosen as the benchmark model. The model is a slightly modified version of the VECM proposed by Rukelj (2010), which is a multivariate model with relatively good forecasting performance compared to the other forecasting methods examined later in this chapter. Since the main purpose of the model is forecasting, a reduced form of the model is considered:

$$\Delta x_t = \alpha\left(\beta' x_{t-1} + \hat{b}t + \hat{c}\right) + \hat{\Gamma}_1 \Delta x_{t-1} + \ldots + \hat{\Gamma}_6 \Delta x_{t-6} + \hat{\delta} d_t + u_t, \tag{1}$$

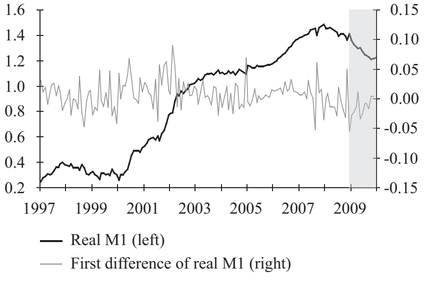
where $t=7, 8, \ldots, T=144$, x_t is a vector of endogenous variables (monetary aggregate M1, government expenditures and index of economic activity), $\hat{b}$ is a vector of estimated deterministic trend coefficients, $\hat{c}$ is a vector of constants, $\hat{\delta}$ is a vector of dummy variables' coefficients and u_t is a vector of a zero mean residuals and a positive definite covariance matrix $\hat{\Sigma}$. The model can be rewritten in a restricted vector auto regression (VAR) form as follows:

$$x_t = \sum_{i=1}^{7} \hat{\Pi}_i x_{t-i} + \hat{\pi}_0 + \hat{\pi}_1 t + \hat{\delta} d_t + u_t, \tag{2}$$

where parameters are restricted to $\hat{\Pi}_1 = I_3 - \alpha\beta' + \hat{\Gamma}_1$, $\hat{\Pi}_i = \hat{\Gamma}_i - \hat{\Gamma}_{i-1}$, $i = 2, ..., 6$, $\hat{\Pi}_7 = -\hat{\Gamma}_6$, $\hat{\pi}_0 = \alpha\hat{c}$ and $\hat{\pi}_1 = \alpha\hat{b}$.

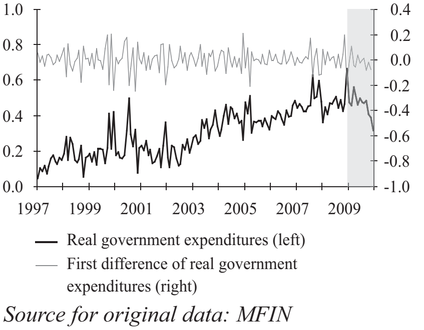
The economic variables considered for this model are, as in Rukelj (2010), real monetary aggregate M1 (figure 1); real government expenditures defined as the sum of expenditure and acquisition of non-financial assets GFS 2001 categories (figure 2); and index of economic activity (figure 3). Index of economic activity was constructed from the available high frequency indicators weighted by their approximate shares in gross value added. The model is estimated using monthly data over the period from January 1997 to December 2008. The data for 2009 are set apart for comparison with the out-of-sample model forecasts. All the data series are rebased to year 1997, transformed to natural logarithms and seasonally adjusted by the Census X12 method.

FIGURE 1
Monetary aggregate M1



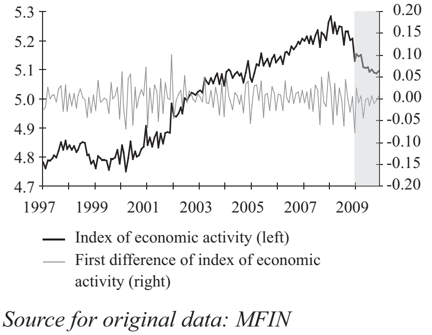
Source for original data: CNB

FIGURE 2
Government expenditures



Source for original data: MFIN

FIGURE 3
Index of economic activity



Source for original data: MFIN

Next, we summarize the results of unit root and cointegration tests. In none of the cases are we able to reject the presence of unit root around a deterministic trend. However, according to the Augmented Dickey-Fuller test, all three series are stationary in their first differences at the one percent significance level. Both trace and maximum eigenvalue tests suggest two cointegrating equations at the five percent significance level. Basically, the VECM is estimated here in a two stage estimation procedure: we first test for cointegration using the Johansen approach, and then estimate the error correction equations with OLS in the second step. The only difference with Rukelj (2010) is that two more dummies (January 2003 and June 2004) were introduced to account for outliers.

Once the benchmark model is estimated, we recovered its residuals for the stochastic simulations. Therefore, we check autocorrelation, normality and heteroscedastic properties of the residuals. The results are displayed in table 1. According to the Portmanteau test for autocorrelation, we do not reject the null hypothesis of no autocorrelation in the first 12 lags at the 10 percent significance level. The autoregressive conditional heteroscedasticity ARCH-LM test on individual residual series does not imply heteroscedasticity problems. The Jarque-Bera test indicates that the residuals from the first (monetary aggregate M1) and the third (index of economic activity) restricted VAR equations are close to normal distribution. Normality is rejected for the residuals from the second equation (government expenditures) in which distribution is skewed to the right.

TABLE 1
Diagnostic tests

Portmanteau test with 12 lags, degrees of freedom: 74				
	Test statistic	P-value		
	89.8514	0.1013		
ARCH-LM test				
	Test statistic	P-value		
u_1	12.3459	0.4183		
u_2	15.1633	0.2326		
u_3	11.9129	0.4527		
Jarque-Bera test				
	Test statistic	P-value	Skewness	Kurtosis
u_1	1.0741	0.5845	0.0517	3.4212
u_2	14.7980	0.0006	0.7451	3.6094
u_3	1.0660	0.5868	0.0603	3.4149

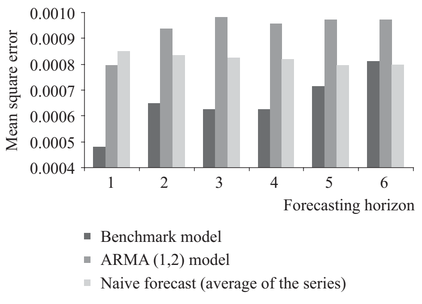
u_1: residuals from the monetary aggregate equation
u_2: residuals from the government expenditures equation
u_3: residuals from the index of economic activity equation
Source: Authors' calculations.

The forecasting performance of the benchmark model is examined by comparing it to a univariate autoregressive integrated moving average (ARIMA) model and to naive forecasts. Forecasts of the first differenced index of economic activity series are considered since the series are found to be integrated of order one. The autocorrelation function and the partial autocorrelation function of the series suggest an autoregressive process of order one and moving average process of order two. Thereafter, we choose an ARIMA (1, 1, 2). The naive forecasts are calculated as a simple average of the subsamples of the considered series.

Out-of-sample forecasts were obtained from the 48 recursively estimated models where the first sub sample starts in January 1997 and ends in December 2005. Estimation period is then extended by one month up to the last subsample, which covers the period from January 1997 to December 2008. For each recursively estimated model six month ahead forecasts are obtained and mean squared errors (MSE) for individual forecasting horizons are calculated. Comparison of the forecasts derived from the two models and naive forecasts (figure 4) indicate that the benchmark model performs better than the other two methods in the six month forecasting horizon.

FIGURE 4

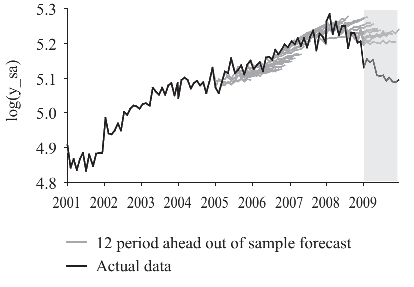
Comparison of the forecasting performance for the first differenced index of economic activity series



Source: Authors' calculations.

FIGURE 5

Out of sample forecasts of the recursively estimated benchmark model for the index of economic activity



Source: Authors' calculations.

The forecasting horizon is also extended to twelve months and the forecasts of the index of economic activity in levels are also observed (figure 5). Analysis of the out-of-sample forecasts in the period 2005 to 2009 shows there is a significant increase of the MSE for the period starting at the end of 2008 as a result of the great shock due to the world economic crisis. This illustrates underperformance of the benchmark model when large shocks hit the economy and hence the inadequacy of point forecasts in this situation. Therefore, there is a clear need for a different forecasting framework, which can account for strong shocks and extreme events. We propose density forecasts which can cover the whole set of possi-

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ble outcomes with probabilities of their realization and in this way account for uncertainty in forecasting.

3 METHODOLOGY

The steps followed in forecasting by the underlying benchmark model, which take uncertainty into consideration, are summarized in this section. Uncertainty was incorporated into the forecasts by two approaches: uncertainty about the future, and uncertainty about the future and the parameters. We use the methodology proposed by Garratt et al. (2003) in the calculation of the density forecasts. When the future uncertainty alone is considered, the density forecasts are generated without parameter uncertainty recursively by stochastic simulations.

Accounting for the shocks is the critical point in density forecasting where a choice of shocks' distribution might be required according to the different types of uncertainty taken into account. In this sense, we use parametric and non-parametric methods for the generation of shocks, while the probability bands are constructed from the obtained set of simulated values. As one set of the residuals from the benchmark model shows a skewed distribution, we also consider a simulation of shocks by skewed distribution to account for asymmetries.

Parametric and non-parametric methods for generation of shocks are also used when both future and parameter uncertainties are considered. To generate density forecasts comprising both uncertainties, a set of new in sample values of the variables is calculated by stochastic simulations using the original benchmark model and generated set of shocks. The model is then re-estimated for each replication of the in sample generated series and future uncertainty is taken into account for each re-estimated model in the final step. Robustness checks are made by considering different numbers of replications in constructing the probability bands.

Once the probability bands are calculated, fan charts are used for the presentation of the calculated density forecasts. In the end, the forecasts are evaluated by checking whether probability integral transforms of actual data resemble the uniform distribution. Non-parametric Kolmogorov-Smirnov and Anderson-Darling tests are used for that purpose. In what follows, the above mentioned steps are explained in detail.

3.1 FUTURE UNCERTAINTY

The methodology of stochastic simulation techniques proposed by Garratt et al. (2003) is applied to the estimated restricted VAR model given by equation (1). This methodology consists of generating the shocks from an assumed distribution which are then used as error terms in the equations of the benchmark model. In this way a set of forecasts is obtained that serves the purpose of probability band calculation. The point forecasts in period h are given by:

$$\hat{x}_{T+h} = \sum_{i=1}^{7} \hat{\Pi}_i \hat{x}_{T+h-i} + \hat{\pi}_0 + \hat{\pi}_1 (T+h) \tag{3}$$

where *h=1, 2, ..., H* is the period of the forecasting horizon, with the initial values $\hat{x}_T$, $\hat{x}_{T-1}$, ... $\hat{x}_{T-6}$ given. The set of forecasts used for the calculation of probability bands is obtained with a stochastic simulation algorithm where each replication of the algorithm is calculated according to the following equation:

$$\hat{x}^{(r)}_{T+h} = \sum_{i=1}^{7} \hat{\Pi}_i \hat{x}^{(r)}_{T+h-i} + \hat{\pi}_0 + \hat{\pi}_1 (T+h) + u^{(r)}_{T+h} \tag{4}$$

where *r=1, 2, ..., R* denotes r^{th} replication of the algorithm, $\hat{x}^{(r)}_T = x_T$, $\hat{x}^{(r)}_{T-1} = x_{T-1}$, ... $\hat{x}^{(r)}_{T-6} = x_{T-6}$ and the term $u^{(r)}_{T+h}$ represents simulated shocks generated with a parametric or non-parametric method.

In the parametric approach, firstly *3*h* random draws $\varepsilon^{(r)}_{T+h}$ are obtained from an assumed probability distribution, which is in this case a multivariate normal distribution with mean zero and covariance matrix *I*. The simulated shocks are then obtained as:

$$u^{(r)}_{T+h} = \hat{P}\varepsilon^{(r)}_{T+h} \tag{5}$$

where $\hat{P}$ is the Cholesky decomposition of estimated covariance matrix $\hat{\Sigma}$, such that $\hat{\Sigma} = \hat{P}\hat{P}'$ and $\varepsilon^{(r)}_{T+h} \approx IIN(0, I)$. In the non-parametric approach, simulated shocks are generated as *R* random draws with replacements from the in-sample residuals u_t.

Risks may be asymmetric, meaning that there is a higher probability of observation being on one side of the mode. If the risks are unbalanced, a distribution of simulated shocks should account for asymmetries. The unbalanced risks can be implied by the skewed distribution of the in-sample residuals but can also represent a subjective view on the risks in the future. To account for asymmetries in the residuals from the government expenditure equation, shocks are generated with the parametric method from a skewed distribution. The shocks are defined as:

$$u^{(r)}_{T+h} = \hat{P}v'^{(r)}_{T+h} \tag{6}$$

where $\hat{P}$ is a decomposition of the estimated covariance matrix $\hat{\Sigma}$ – as defined earlier. The first and third column of $v'^{(r,s)}_{T+h}$ are identical to the first and third column of $\varepsilon^{(r)}_{T+h}$ while the second element $\left(w^{(r)}_{T+h}\right)$ is generated by a two-piece normal distribution[4] as follows:

[4] See Tay and Wallis (2000) for more details.

$$f(w_{T+h}^{(r)},\mu,\sigma_1,\sigma_2)=\begin{cases} C\exp\left\{-\dfrac{1}{2\sigma_1^2}\left(w_{T+h}^{(r)}-\mu_{T+h}\right)^2\right\}, for \rightarrow w_{T+h}^{(r)} \leq \mu_{T+h} \\ C\exp\left\{-\dfrac{1}{2\sigma_2^2}\left(w_{T+h}^{(r)}-\mu_{T+h}\right)^2\right\}, for \rightarrow w_{T+h}^{(r)} > \mu_{T+h} \end{cases} \tag{7}$$

where $C = k(\sigma_1 + \sigma_2)^{-1}$ and $k = \sqrt{2/\pi}$. Skewness is described by $\gamma = \bar{\mu} - \mu =$ $= \mathrm{k}(\sigma_2 - \sigma_1)$ where $\bar{\mu}$ is the mean and μ is the mode of distribution.

After the set of simulated shocks is obtained, probability bands are defined by the threshold values, which are calculated in the following way:

- b^h denotes a vector of $\hat{x}_{T+h}^{(r)}$ values such that $b^h(1) < b^h(2) \ldots < b^h(R)^h$,
- the upper threshold value of the *p%* probability band centered over the value $b^h(R/2)$ in the forecasting period *T+h* is defined as the element of the vector b^h(i) where $i = \frac{R}{2}(1+p)$ and $0 < p < 1$,
- the lower threshold value of the *p%* probability band centered over the value $b^h(R/2)$ in the forecasting period *T+h* is defined as the element of the vector b^h(j) where $j = \frac{R}{2}(1-p)$ and $0 < p < 1$.

3.2 FUTURE AND PARAMETER UNCERTAINTY

In this case, a bootstrap procedure is used to simulate *S* in sample values of $\hat{x}_t$ such that:

$$\hat{x}_t^{(s)} = \sum_{i=1}^{7} \hat{\Pi}_i \hat{x}_{t-i}^{(s)} + \hat{\pi}_0 + \hat{\pi}_1 t + \hat{\delta} d_t + u_t^{(s)} \tag{8}$$

where *t=8, 9, ..., T=144*, and *s=1, 2, ..., S* denotes s^{th} replication of the algorithm, with initial values $\hat{x}_1$, $\hat{x}_2$, ... $\hat{x}_7$ given and $u_t^{(s)}$ representing simulated shocks generated as *S* random draws with replacements from the in sample residuals u_t.

The following step is the estimation of the VAR model *S* times to obtain $\hat{\Pi}^{(s)}$, $\hat{\pi}_0^{(s)}$, $\hat{\pi}_1^{(s)}$ and $\hat{\Sigma}^{(s)}$, where $\hat{\Sigma}^{(s)}$ is the residuals' covariance matrix of the s^{th} estimated model. For each estimated model, *R* replications of the forecasts for the period *T+h* are calculated as:

$$\hat{x}_{T+h}^{(r,s)} = \sum_{i=1}^{7} \hat{\Pi}_i^{(s)} \hat{x}_{T+h-i}^{(r,s)} + \hat{\pi}_0^{(s)} + \hat{\pi}_1^{(s)}(T+h) + u_{T+h}^{(r,s)} \tag{9}$$

As in the previous case, threshold values of the probability bands were calculated using the following formulas:

- b^h denotes a vector of $\hat{x}_{T+h}^{(r,s)}$ values such that $b^h(1) < b^h(2) \ldots < b^h(R*S)^h$,

- the upper threshold value of the $p\%$ probability band centered over the value $b^h\ (R{*}S/2)$ in the forecasting period $T+h$ is defined as the element of the vector $b^h\ (i)$ where $i = \frac{R*S}{2}(1+p)$ and $0 < p < 1$,
- the lower threshold value of the $p\%$ probability band centered over the value $b^h\ (R{*}S/2)$ in the forecasting period $T+h$ is defined as the element of the vector $b^h\ (j)$ where $j = \frac{R*S}{2}(1-p)$ and $0 < p < 1$.

3.3 FAN CHARTS

The usual way of presenting density forecasts is the fan chart. For each forecasting period, a fan chart depicts bands within which the forecasted variable will fall with a given probability. Probability bands are presented for probability clusters of equal size and are illustrated with a lighter color as they move away from the central projection. Likewise, probability bands become wider as they move away from the central projection and with an increasing forecasting horizon as uncertainty increases.

The starting point is an obtained set of density forecasts for each period of the forecasting horizon. That is, in each forecasting period there is a probability distribution for the variable of interest. In the symmetric case the mean, median and mode of this probability distribution are equal and represent the central projection. Otherwise, it would correspond to the asymmetric distribution and represent a scenario in which risks would be unbalanced.

For both symmetric and asymmetric distributions, fan chart bands are obtained by calculating nine deciles of the forecasted variables, where the first ten percent of the variable forecasts closest to the mode are represented by a darker and more solid color, followed by the next decile represented with a lighter and less solid color, and so on until ninety percent of the variable forecasts is covered. The final band in the fan chart is implicitly taken as the white area and it represents the opened interval for which there is a ten percent of chance that forecasts will fall in.

3.4 FORECAST EVALUATION

An evaluation of forecasts is carried out by Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests of probability integral transforms (PITs) of actual realizations. A PIT associates the actual realization in the observed period with the probability implied by the density forecasts that this realization will be equal or less than actually observed.

The PITs for twelve step ahead density forecasts are tested for the 36 in-sample realizations starting from year 2005 for both cases, with and without allowing for parameter uncertainty. The hypothesis that these PITs are random draws from a uniform distribution *U(0,1)* is then tested by calculating the KS statistic, such that large values are indicative of significant departures of the sample cumulative density function from the hypothesized uniform distribution. Since density forecasts imply equal probability of realization in each probability band, good density forecasts would result in a uniformly distributed PIT of actual realizations.

The KS test is a fully non-parametric test for comparing two probability distributions. It is robust as it does not rely on the location of the mean and does not depend on the underlying cumulative distribution function being tested. Unlike chi-square goodness-of-fit test, the KS test does not depend on sample size for the approximations to be valid and it is an exact test.

Formally, the following null hypothesis was tested by the KS test:

$$H_0 : F(z_i) = F_u(z_i) \tag{11}$$

$$H_1 : F(z_i) \neq F_u(z_i) \tag{12}$$

where z_i denotes the set of ordered PITs of observed realizations in forecasting period i, $F(z_i)$ is unknown cumulative probability distribution function and $F_u(z_i)$ represents a uniform cumulative probability distribution function such that $F_u(z_i)=z_i$. The KS test statistic D is defined by:

$$D = \sup_z \left| \hat{F}(z_i) - F_u(z_i) \right| \tag{13}$$

where $\hat{F}(z)$ defines the empirical cumulative distribution function as:

$$\hat{F}(z_i) = \frac{(\#(z_i) \leq z_i)}{n} \tag{14}$$

Some important and very well known limitations of the KS test are: (1) it only applies to continuous distributions; (2) it tends to be more sensitive near the center of the distribution than at the tails; and (3) a distribution must be fully specified, i.e., if location, scale, and shape parameters are estimated from the data, the critical region of the KS test is no longer valid and it typically must be determined by simulation[5]. Due to limitations 2 and 3 above, we also perform the AD goodness-of-fit test.

Although in the AD goodness-of-fit test both null and alternative hypothesis are as in equations (11) and (12), it is more sensitive to deviations in the tails of the distribution than the KS test. This is because the test consists on calculating the AD statistic (A^2) defined as:

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \left[\ln F_u(z_i) + \ln\left(1 - F_u(z_{n-i+1})\right) \right] \tag{15}$$

where n is the sample size, in our case equal to 36.

[5] See Sheskin (2000) for more details.

While in the KS test the minimum and maximum values of the theoretical uniform distribution function are taken as given *(0, 1),* in the AD test these parameters are estimated by maximum likelihood. Estimated minimum and maximum values of the theoretical uniform distribution function used in the AD test are referred to as parameter A and parameter B later in the text.

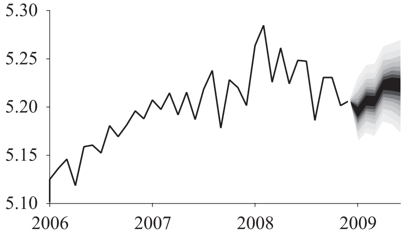
4 RESULTS

Five experiments are carried out by using the outlined methodology to examine different effects of various assumptions on the density forecasts. Three experiments include future uncertainty alone while in other two both future and parameter uncertainty are considered. Robustness checks indicate no significant changes in calculated probability bands at 500 replications in the first three experiments and 750 replications in the last two experiments. Density forecasts from all experiments are presented in the form of fan charts and evaluated by the KS and AD tests. All the estimations and calculations are coded and performed in Gauss, and also some procedures available in J-Multi package are used. The KS and AD tests of PITs of actual realizations are performed in EViews.

The first experiment examines density forecasts when the future uncertainty alone is considered with parametric simulated shocks and symmetric risks. Up to 1,000 shocks were generated by the simulation algorithm. The results of this experiment are presented in figures 6 and 7, and table 2. As expected, probability bands of the fan chart are the narrowest in the central probability band and get wider with increasing distance from the central forecast. Also, the probability bands widen with the forecasting horizon.

FIGURE 6

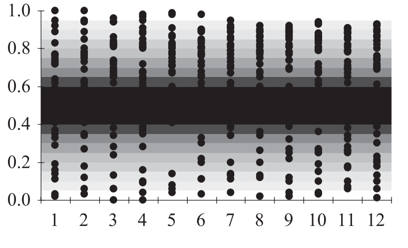
Fan chart 1: future uncertainty, parametric simulated shocks, symmetric risks



Source: Authors' calculations.

FIGURE 7

PITs of realizations: future uncertainty, parametric simulated shocks, symmetric risks



Source: Authors' calculations.

The black dots in figure 7 represent the PITs of realizations in each period of the forecasting horizon and their positions on faded regions indicate in which percentile of the fan chart they fall in. A general overview of PITs of 36 in-sample realizations indicates their even distribution in the interval from one to zero for most of the forecasting horizon. Formal evaluation of the density forecasts from the first experiment by both tests indicates a better forecasting performance in the first six periods of the forecasting horizon. There is decrease of numbers of realizations at the highest decile as the forecasting horizon increases (see figure 7), the same result is also suggested when evaluating by the AD test.

Table 2
Future uncertainty, parametric simulated shocks and symmetric risks test

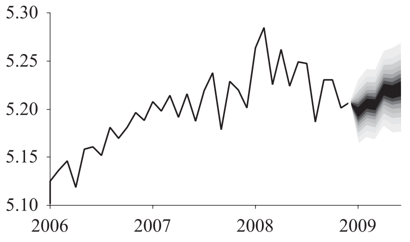
Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
KS test results												
Value (D)	0.14	0.20	0.24	0.18	0.30	0.26	0.21	0.24	0.22	0.22	0.21	0.19
Probability	0.49	0.09	0.03	0.16	0.00	0.01	0.07	0.02	0.06	0.04	0.08	0.13
AD test results												
Value (A2)	0.97	1.48	3.28	2.21	4.14	4.10	4.02	6.17	5.14	3.49	2.69	2.16
Probability	0.37	0.18	0.02	0.07	0.01	0.01	0.01	0.00	0.00	0.02	0.04	0.07
Parameter (A)	0.02	0.03	0.00	0.00	0.04	0.03	0.04	0.02	0.02	0.03	0.06	0.01
Parameter (B)	1.00	1.00	0.96	0.98	0.99	0.98	0.95	0.92	0.92	0.94	0.91	0.93
Standard error	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02

Source: Authors' calculations.

The second experiment is designed to consider the future uncertainty with 1,000 shocks generated by the non-parametric method. In the non-parametric method simulated shocks are generated as random draws with replacements from the in-sample residuals. This experiment resulted in density forecasts with features very similar to those of the first experiment, which can be seen in figures 9 and 10. The KS and AD test results shown in table 3 also indicate a better forecasting performance in the first six periods of the forecasting horizon. There is also an evident lack of realizations with PITs higher than 0.95 in the second part of the forecasting horizon.

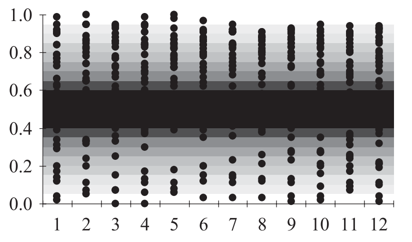
The future uncertainty with 1,000 parametric simulated shocks and asymmetric risks is examined in the third experiment. The diagnostic tests confirm the skewness of the distribution of the residuals from the government expenditure equation. Therefore, the shocks for the second VECM equation were generated by the two-piece normal distribution with $\sigma_1 = 1$ and $\sigma_2 = 1.75$. The results from the third experiment are presented in figures 10 and 11, and table 4. The fan chart shows features similar to those from the previous two experiments. The PITs of realizations in the first six periods of the forecasting horizon seem more evenly distributed than in the previous two experiments. This is corroborated by the formal KS and AD test where for all six periods the null hypothesis of uniform distribution could not be rejected at the one percent significance level. The tests also indicated uniformly distributed data in the second six periods of the forecasting horizon. Nevertheless, parameter B estimated by the AD test is significantly lower than 1, indicating lack of realizations with PITs higher than 0.9.

Figure 8
Fan Chart 2: future uncertainty, non-parametric simulated shocks



Source: Authors' calculations.

Figure 9
PITs of realizations: future uncertainty, non-parametric simulated shocks



Source: Authors' calculations.

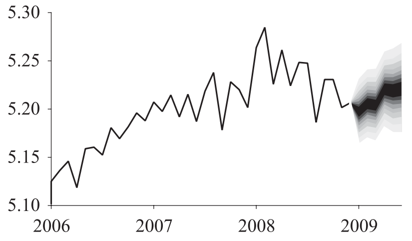
Table 3
Future uncertainty, non-parametric simulated shocks and symmetric risks test

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
KS test results												
Value (D)	0.17	0.22	0.26	0.22	0.29	0.28	0.24	0.28	0.25	0.24	0.22	0.20
Probability	0.21	0.06	0.01	0.05	0.00	0.01	0.02	0.00	0.02	0.03	0.05	0.09
AD test results												
Value (A2)	1.14	1.60	5.27	3.25	4.30	5.49	5.75	7.19	6.17	4.21	2.56	3.40
Probability	0.29	0.15	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.01	0.05	0.02
Parameter (A)	0.02	0.05	0.00	0.00	0.06	0.03	0.03	0.03	0.01	0.02	0.07	0.01
Parameter (B)	0.99	1.00	0.95	0.99	1.00	0.97	0.95	0.91	0.93	0.95	0.94	0.94
Standard error	0.03	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02

Source: Authors' calculations.

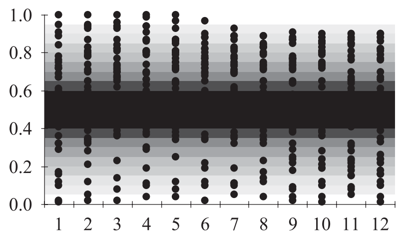
Both future and parameter uncertainties in the density forecast are examined in the fourth and the fifth experiment. The following procedure is used to obtain the set of forecasts for calculation of probability bands: (1) the initial 7 lags are taken from the original series; (2) the set of shocks is obtained by parametric and non-parametric methods; (3) the forecasts are calculated by the initially estimated model by applying a shock to each observation in each in-sample period; (4) a 1,000 samples with three variables are obtained as a result and 1,000 models were estimated from these samples; (5) forecasts are calculated as in the case of future uncertainty based on these 1,000 models.

FIGURE 10
Fan chart 3: future uncertainty, parametric simulated shocks, asymmetric risks



Source: Authors' calculations.

FIGURE 11
PITs of realizations: future uncertainty, parametric simulated shocks, asymmetric risks



Source: Authors' calculations.

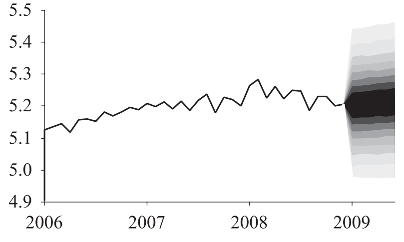
TABLE 4
Future uncertainty, parametric simulated shocks and asymmetric risks test

Period	1	2	3	4	5	6	7	8	9	10	11	12
KS test results												
Value (D)	0.13	0.20	0.27	0.20	0.28	0.22	0.18	0.18	0.13	0.15	0.10	0.10
Probability	0.53	0.09	0.01	0.09	0.01	0.06	0.16	0.17	0.58	0.37	0.85	0.85
AD test results												
Value (A2)	0.87	1.63	3.77	2.02	3.91	3.35	2.24	2.58	1.63	1.86	0.50	0.55
Probability	0.44	0.15	0.01	0.09	0.01	0.02	0.07	0.05	0.15	0.11	0.75	0.69
Parameter (A)	0.01	0.02	0.02	0.04	0.04	0.02	0.05	0.04	0.02	0.01	0.05	0.01
Parameter (B)	1.00	1.00	1.00	1.00	1.00	0.97	0.93	0.89	0.91	0.90	0.90	0.90
Standard error	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02

Source: Authors' calculations.

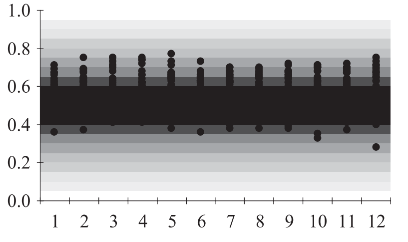
Density forecasts incorporating future and parameter uncertainty with shocks generated by the parametric method are also investigated in the fourth experiment. The fan chart shown in figure 12 has much wider probability bands than the fan charts from the experiments where future uncertainty only was considered, which is an expected result, given the characteristics of this particular experiment. Probability bands also get wider as they move away from the mode and the forecasting horizon increases. Most of the PITs of realizations are concentrated in the interval between 0.3 and 0.7. The large width of the probability bands is confirmed by the results of the formal tests given in table 5. The KS test null hypothesis of uniform distribution of PITs of actual realizations is rejected for all the periods in the forecasting horizon at the one percent level of significance.

FIGURE 12
Fan chart 4: future and parameter uncertainty, parametric simulated errors, symmetric risks



Source: Authors' calculations.

FIGURE 13
PIT of realizations: future and parameter uncertainty, parametric simulated shocks, symmetric risks



Source: Authors' calculations.

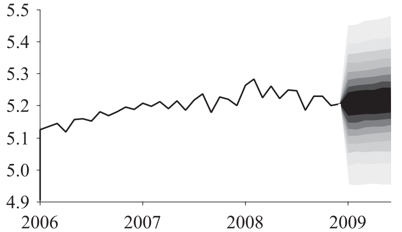
TABLE 5
Future and parameter uncertainty, parametric simulated shocks and symmetric risks test

Period	1	2	3	4	5	6	7	8	9	10	11	12
KS test results												
Value (D)	0.41	0.42	0.41	0.43	0.40	0.42	0.39	0.38	0.38	0.37	0.38	0.37
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AD test results												
Value (A2)	3.90	3.13	2.13	1.65	2.62	5.57	3.86	4.66	1.99	4.47	2.39	4.16
Probability	0.01	0.02	0.08	0.14	0.04	0.00	0.01	0.00	0.09	0.01	0.06	0.01
Parameter (A)	0.36	0.37	0.41	0.41	0.38	0.36	0.38	0.38	0.38	0.33	0.37	0.28
Parameter (B)	0.71	0.75	0.75	0.75	0.77	0.73	0.70	0.70	0.72	0.71	0.72	0.75
Standard error	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Source: Authors' calculations.

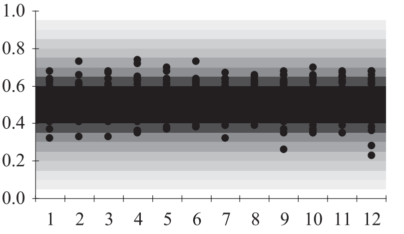
The fifth experiment examined density forecasts with incorporated future and parameter uncertainty and shocks generated by the non-parametric method. The results of this experiment presented in figures 14 and 15, and table 6 are almost identical to the previous experiment. They basically present quite wide probability bands and lack of observations in the six marginal probability bands of the fan chart.

Figure 14
Fan chart 5: future and parameter uncertainty, non-parametric simulated shocks



Source: Authors' calculations.

Figure 15
PIT of realizations: future and parameter uncertainty, non-parametric simulated shocks



Source: Authors' calculations.

Table 6
Future and parameter uncertainty, non-parametric simulated shocks test

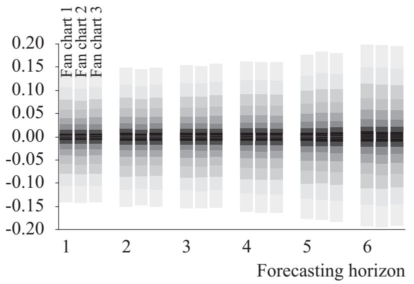
Period	1	2	3	4	5	6	7	8	9	10	11	12
KS test results												
Value (D)	0.35	0.38	0.38	0.35	0.39	0.38	0.36	0.39	0.33	0.35	0.35	0.32
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AD test results												
Value (A^2)	2.89	4.62	1.45	3.09	2.62	3.52	2.44	1.05	2.35	1.20	1.45	3.51
Probability	0.03	0.00	0.19	0.02	0.04	0.02	0.05	0.33	0.06	0.27	0.19	0.02
Parameter (A)	0.37	0.41	0.41	0.36	0.38	0.38	0.39	0.41	0.35	0.35	0.35	0.28
Parameter (B)	0.68	0.73	0.68	0.74	0.70	0.73	0.67	0.66	0.68	0.70	0.68	0.68
Standard error	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Source: Authors' calculations.

As the fan charts from the first three experiments and the last two experiments are visually very similar, it is difficult to compare them based on the standard fan chart representations. Therefore, comparisons are also made in the figures 16 and

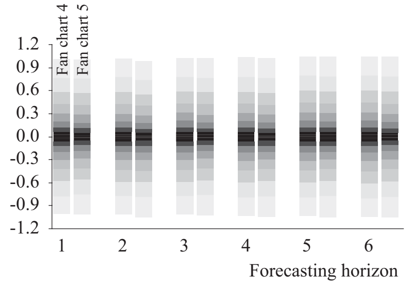
17. The first figure depicts fan charts from the first three experiments and the second figure from the last two experiments as if they had been rotated around the vertical axis by ninety degrees. Probability bands are expressed as deviations from the mode for the six periods of the forecasting horizon. As can be seen, there is a very close similarity within the two groups of fan charts. This could be explained by the fact that in sample residuals are very close to the normal distribution. One clear distinctive feature between the two groups is that probability bands become much wider with an increasing forecasting horizon in the first relative to the second group.

FIGURE 16
Probability bands expressed as deviations from the mode



Source: Authors' calculations.

FIGURE 17
Probability bands expressed as deviations from the mode



Source: Authors' calculations.

5 FURTHER DISCUSSION

Forecasting of economic variables is a crucial component in the economic policy decision making process. Economic activity is inevitably in the focus of forecasting due to its importance in fiscal and monetary policy. The budget preparation process relies heavily on the forecasts of economic activity as this is the main variable for calculating the expected revenue collection. Monetary policy on the other hand uses forecasts of economic activity as the main variable in determining money demand, which is especially important in inflation-targeting regimes. Therefore, the uncertainties in economic activity forecasts should be well understood, accounted for and incorporated in overall economic policy decision making process.

There is no better practical example of the importance of accounting for uncertainties than the situation of the financial and economic crisis in 2009. This was the period of the highest uncertainty in the last decade and a half in the case of Croatia. As shown in figure 5, point forecasts based on the benchmark model and data up to the end of 2008 would be highly imprecise and largely misleading. Although other more complex models where this forecast error would be lower most probably exist, estimated for the same period and data they would hardly be able to account in their point forecasts for the extreme shocks occurred in 2009.

The actual levels of the economic activity index during 2009 also differ from the model results if only future uncertainty is taken into consideration. All the realizations in the first six months of 2009 have PITs lower than 0.05. That is due to a decrease of 8% in the economic activity itself, which reached in mid 2009 levels that were below what is observed in the beginning of 2006. Although the forecasts are not accurate in this period, this is the result of the already mentioned historically greatest economic shock in 2009. In the context of this paper, such a shock would have been observed as if realization was pushed to the last decile of the density forecast.

When the parameter uncertainty was considered for the in-sample forecasts, the main findings were significantly wider probability bands. Calculation of PITs of realizations for the first six months of 2009 offers slightly different results. Namely, PITs are distributed within the range of 0.2 to 0.4, which is an expected result taking into account that the estimation of the model on the sample including data for the year 2009 has a significant impact on the estimated parameters of the model. The main implication is that adding parameter uncertainty to density forecasts might be reasonable when extreme shocks, not yet seen in data, are expected.

6 CONCLUSIONS

Although the economic theory has rapidly developed sophisticated and reasonably well behaved forecasting models, risks and uncertainty are recurrently present. Thus, the need for taking these factors into account when forecasting is what motivates economists to incorporate probabilities to represent this degree of ignorance regarding future events that might affect the economy.

In this paper, we presented the incorporation of the framework to incorporate uncertainties proposed by Garrat et al. (2003) into the forecasts of an economic activity index for Croatia. Using the VECM proposed by Rukelj (2010) as the benchmark model, we allowed the proposed framework to consider future as well as future and parameter uncertainty. Furthermore, we provided a presentation of density forecasts using stochastic simulations of the random shocks with parametric and non-parametric approaches, and the evaluation of density forecasts, the latter using Kolmogorov-Smirnov and Andersen-Darling tests of probability integral transforms.

We found that forecasts of the economic activity index for Croatia are similar when incorporating future uncertainty, regardless of the approach used for shock simulation. Incorporating parameter uncertainty on the other hand, not only resulted in much wider probability bands, but also performed poorly regardless of the approach used for shock simulation. However, the crisis period puts shows the need to account for a broader spectrum of uncertainties. Wider probability bands would then be expected and incorporation of parameter uncertainty would be justified.

The application of the provided forecasting approach could certainly contribute to transparency regarding the views of monetary and fiscal authorities. We expect all the aforementioned, to initiate an important debate among economists and contribute to an important jump in developing and improving this forecasting approach for this economy. Future research we propose should incorporate model uncertainty and additional goodness of fit tests.

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